Static Flexure of Bi-Material Beams and Plates

COURSE PROJECT REPORT

(of ME-759)

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1. Contents

2. Abstract

In this report, static flexural behaviour of slender beams and thin plates made of only linear elastic isotropic material as well as beams and plates made of linear elastic isotropic material with layer of hyper-elastic material of Neo-Hookean type is presented. Rectangular beam is having simply-supported boundary conditions on both ends; square plate is simply-supported on all four edges. Both beam and plate are under the action of uniformly distributed transverse load. Maximum transverse deflections in above mentioned cases are obtained using three-dimensional finite element simulations in Abaqus. These values are compared with corresponding results available in the literature where ever present.

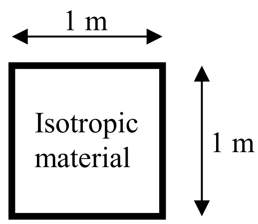
3. Introduction

Beams and plates deform under the action of transverse loading. Shear deformation effects are one of the prominent physical phenomena which govern mechanical behaviour of beams and plates. Theoretical formulations exist for predicting deformation behaviour of beams and plates made of linear elastic isotropic material and hence generally 3D finite element simulation is not required for these cases. However, when such structures have material other than linear elastic isotropic material deformation behaviour is very difficult to obtain theoretically. 3D finite element simulation is the most suitable choice in this case.

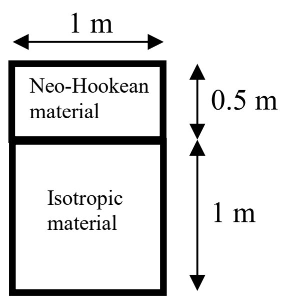
In this report, static flexural behaviour of slender beams and thin plates made of only linear elastic isotropic material as well as beams and plates made of linear elastic isotropic material with layer of hyper-elastic material of Neo-Hookean type is presented. Rectangular beam is having simply-supported boundary conditions on both ends; square plate is simply-supported on all four edges. Both beam and plate are under the action of uniformly distributed transverse load. Maximum transverse deflections in above mentioned cases are obtained using three-dimensional finite element simulations in Abaqus. These values are compared with corresponding results available in the literature where ever present. Effect of changing coefficients of Neo-Hookean material on static flexure of such beams and plates is studied.

4. Problem Statement

Example 1.1: Slender isotropic beam with beam thickness ratio 1/100 and transverse uniformly distributed load 0.001 N/m with cross-sectional details as follows:



Example 1.2: Slender bi-material beam with beam thickness ratio 1/100 and transverse uniformly distributed load 0.001 N/m with cross-sectional details as follows:

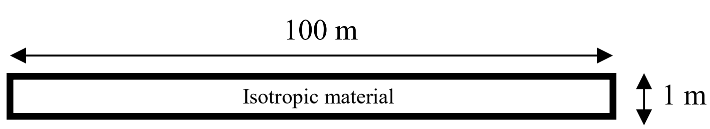


Example 1.2.1: Neo-Hookean material properties C_{10}=10 MPa, D_1=0.

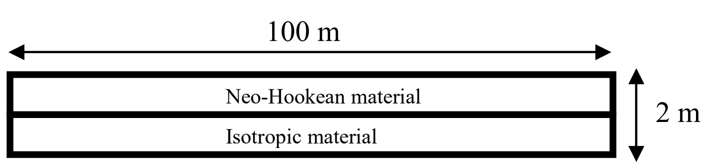
Example 1.2.2: Neo-Hookean material properties C_{10}=50 MPa, D_1=0.

Example 1.2.3: Neo-Hookean material properties C_{10}=100 MPa, D_1=0.

Example 2.1: Thin isotropic plate with plate thickness ratio 1/100 and transverse uniformly distributed load 0.001 N/m^2 with cross-sectional details as follows:



Example 2.2 Thin bi-materal plate with plate thickness ratio 1/100 and transverse uniformly distributed load 0.001 N/m^2 with cross-sectional details as follows:

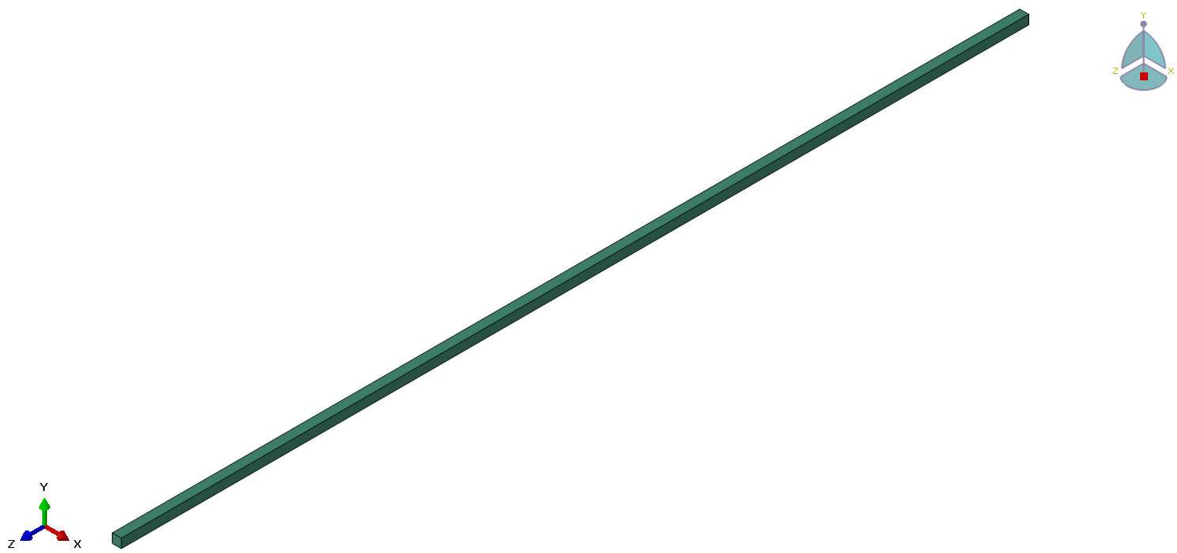


Example 2.2.1: Neo-Hookean material properties C_{10}=10 MPa, D_1=0.

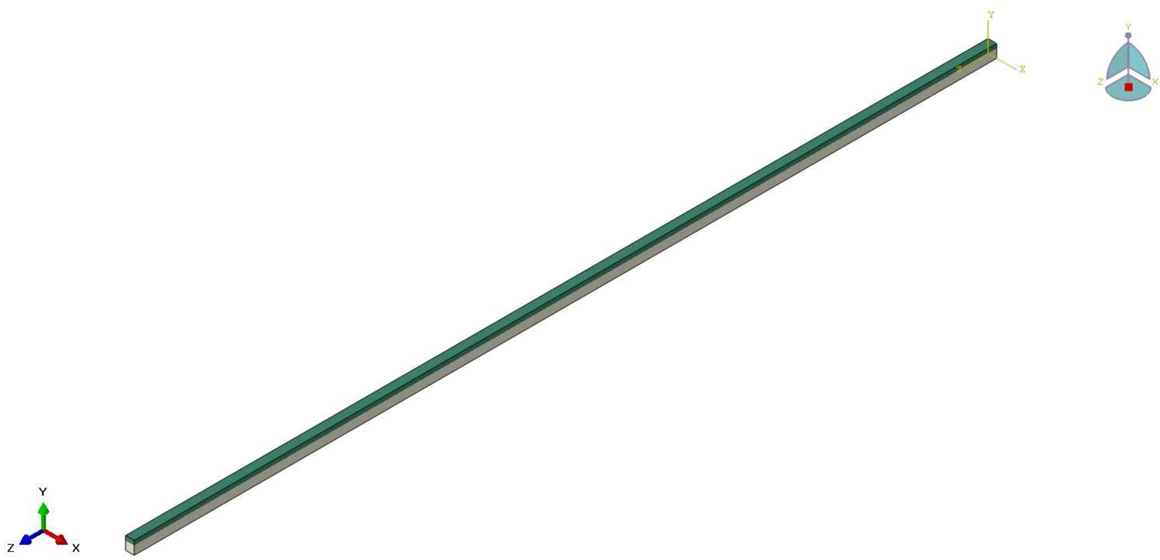
Example 2.2.2: Neo-Hookean material properties C_{10}=50 MPa, D_1=0.

Example 2.2.3: Neo-Hookean material properties C_{10}=100 MPa, D_1=0.

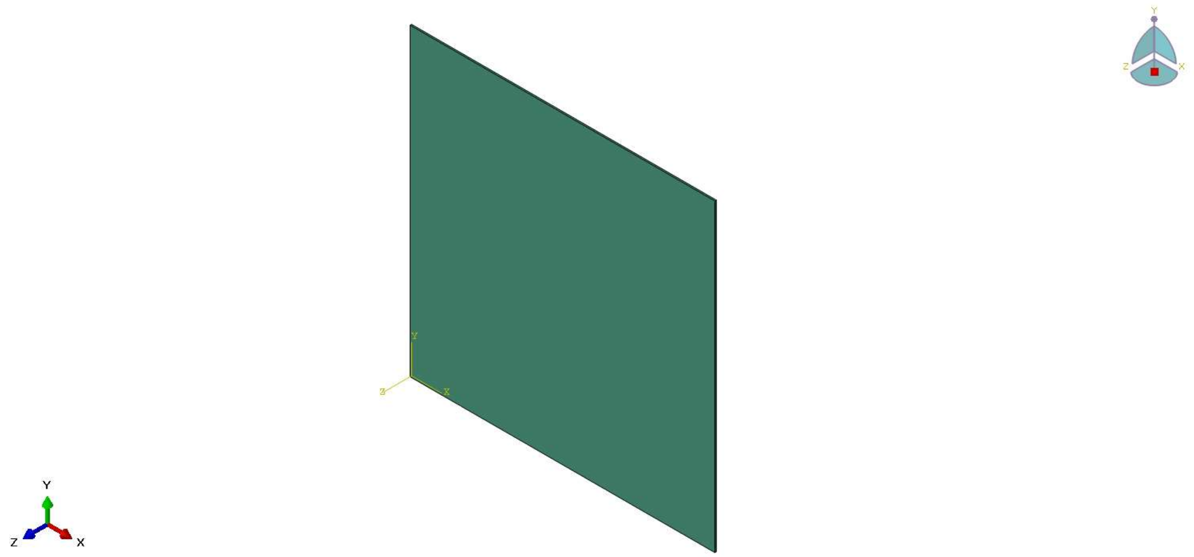
5. Geometry for examples under consideration



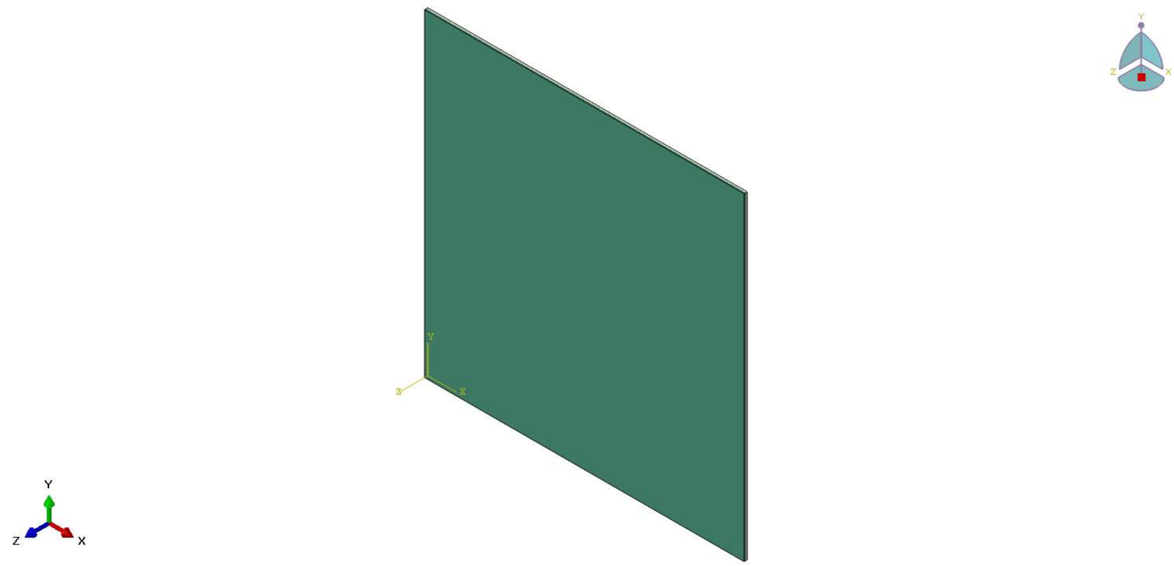
Example 1.1



Example 1.2



Example 2.1



Example 2.2

6. Material Properties

UDL - Notepad

File Edit Format View Help

```
2557569,2557570,2557571,2557572,2557573,2557574,2557575,2557576,2557577,2557578,2557579,2557580,2557581,
2557582,2557583,2557584
2557585,2557586,2557587,2557588,2557589,2557590,2557591,2557592,2557593,2557594,2557595,2557596,2557597,
2557598,2557599,2557600
*Surface, type=ELEMENT, name=Surf-1
_Surf-1_S3, S3
*End Assembly
**
** MATERIALS
**
*Material, name=Steel
*Elastic
 2.1e+11, 0.3
** ----------------------------------------------------------------
**
```

For examples 1.1 and 2.1

BMB1 - Notepad

File Edit Format View Help

```
149998, 149999, 150000
*Surface, type=ELEMENT, name=Surf-1
_Surf-1_S5, S5
*End Assembly
**
** MATERIALS
**
*Material, name=Rubber
*Hyperelastic, neo hooke
 1e+07,0.
*Material, name=Steel
*Elastic
 2.1e+11, 0.3
** ----------------------------------------------------------------
**
```

For examples 1.2 and 2.2

7. Type of Element

BMB1 - Notepad

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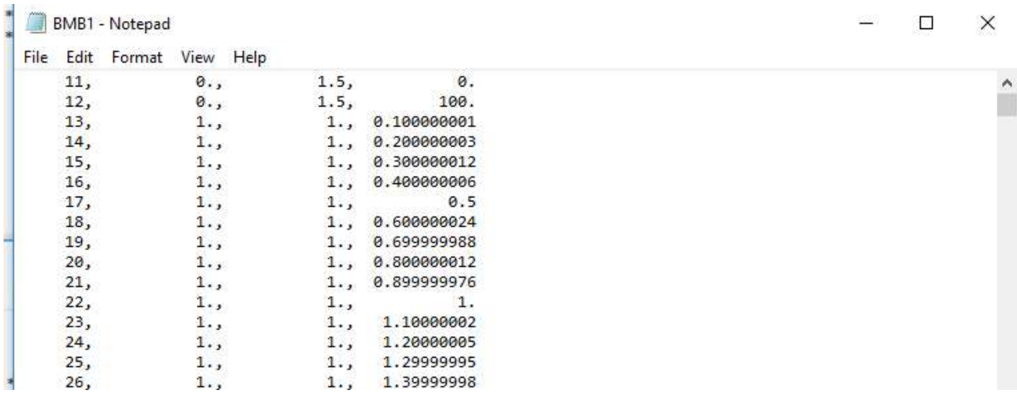
```
 176174,  0.100000001,   1.39999998,  0.300000012
 176175,  0.100000001,   1.39999998,  0.200000003
 176176,  0.100000001,   1.39999998,  0.100000001
*Element, type=C3D8RH
      1,   4062,  24257,  59294,  24095,      1,     13,   6113,   2028
      2,  24257,  24258,  59295,  59294,     13,     14,   6114,   6113
      3,  24258,  24259,  59296,  59295,     14,     15,   6115,   6114
```

For examples 1.1, 1.2, 2.1 and 2.2

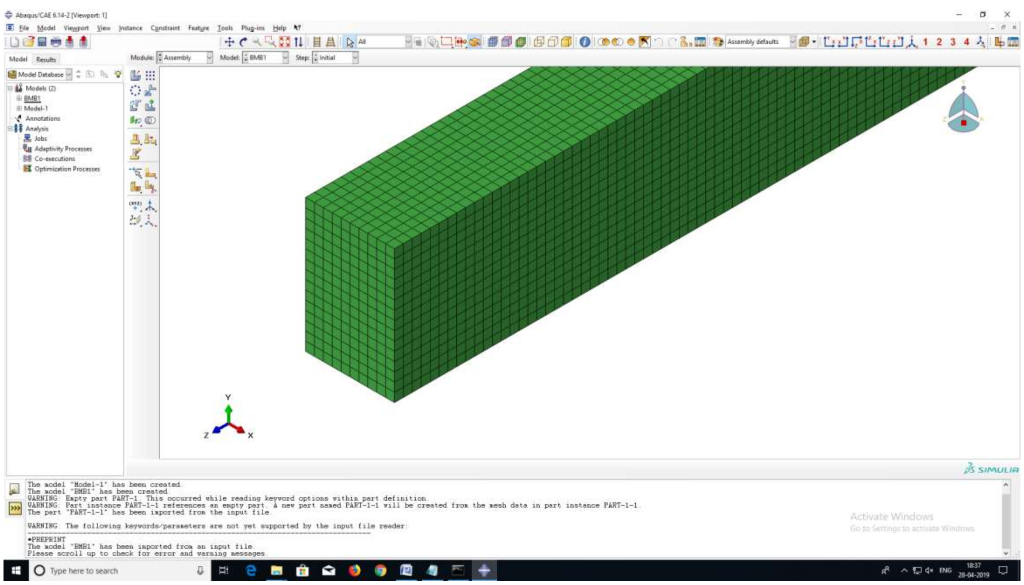
8. Mesh size

For examples 1.1, 1.2, 2.1 and 2.2; mesh size is 0.1


BMB1 - Notepad

File Edit Format View Help

11,	0.,	1.5,	0.
12,	0.,	1.5,	100.
13,	1.,	1.,	0.100000001
14,	1.,	1.,	0.200000003
15,	1.,	1.,	0.300000012
16,	1.,	1.,	0.400000006
17,	1.,	1.,	0.5
18,	1.,	1.,	0.600000024
19,	1.,	1.,	0.699999988
20,	1.,	1.,	0.800000012
21,	1.,	1.,	0.899999976
22,	1.,	1.,	1.
23,	1.,	1.,	1.10000002
24,	1.,	1.,	1.20000005
25,	1.,	1.,	1.29999995
26,	1.,	1.,	1.39999998

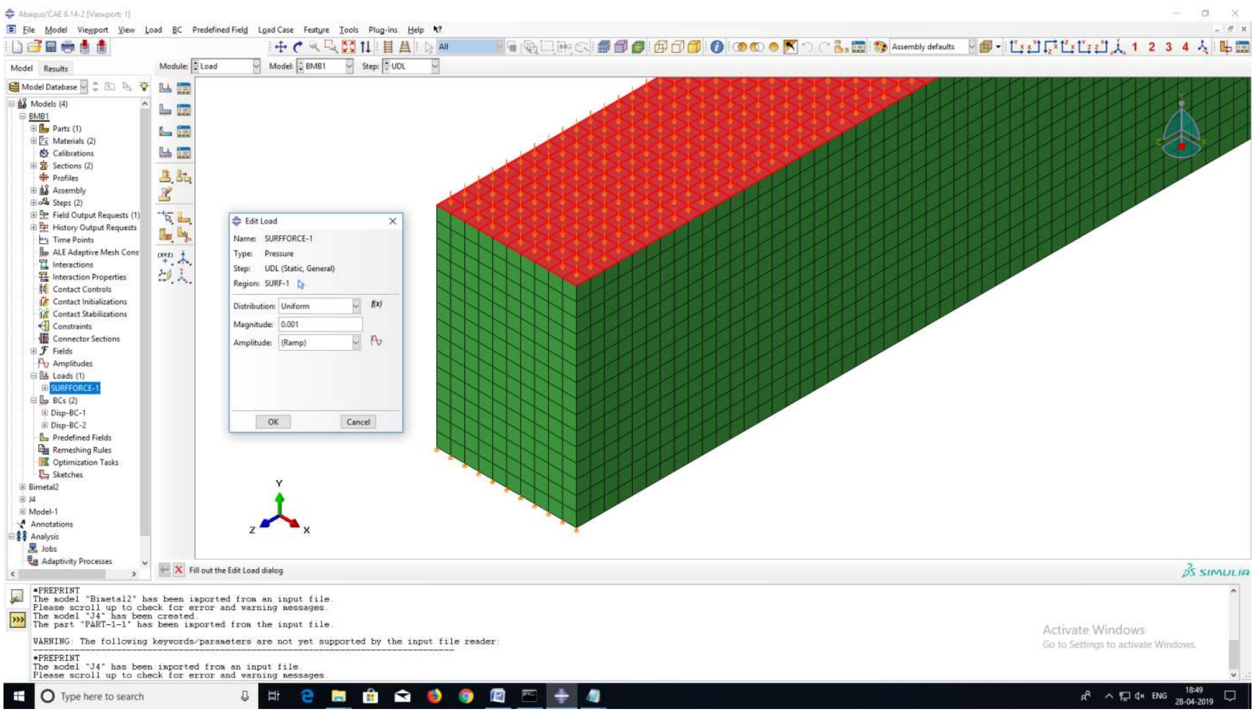


E.g., Examples 1.1 and 1.2

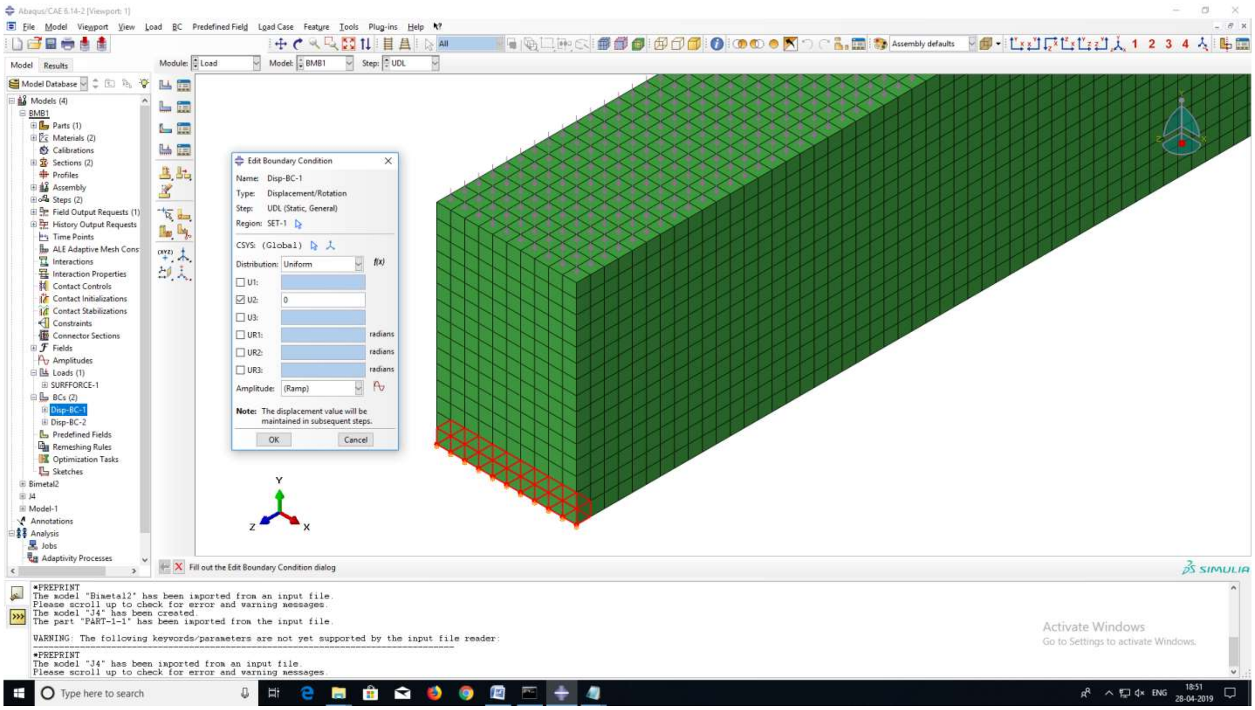
9. Assigning Loads and Boundary conditions

1. In examples 1.1 and 1.2, beam is loaded by uniformly distributed transverse load (UDL). In examples 2.1 and 2.2, plate is loaded by uniformly distributed transverse load.

2. Beam is simply supported in both the ends. Plate is simply supported on all four edges.



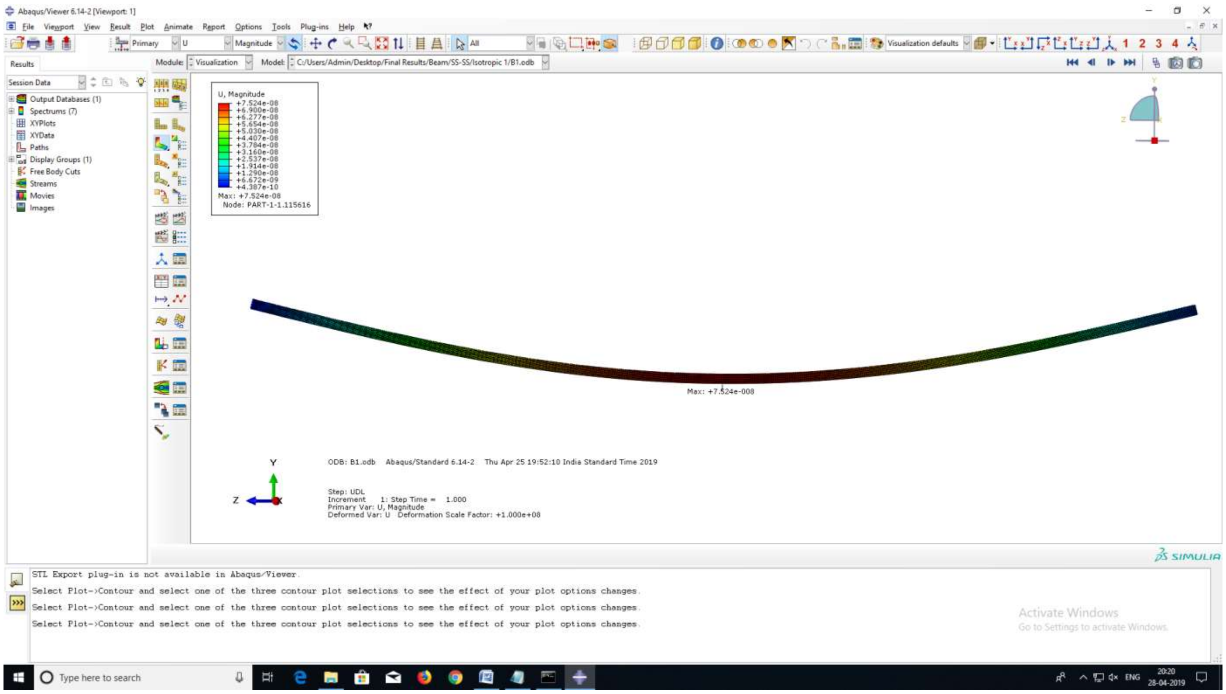
E.g., Examples 1.1 and 1.2 with applied UDL



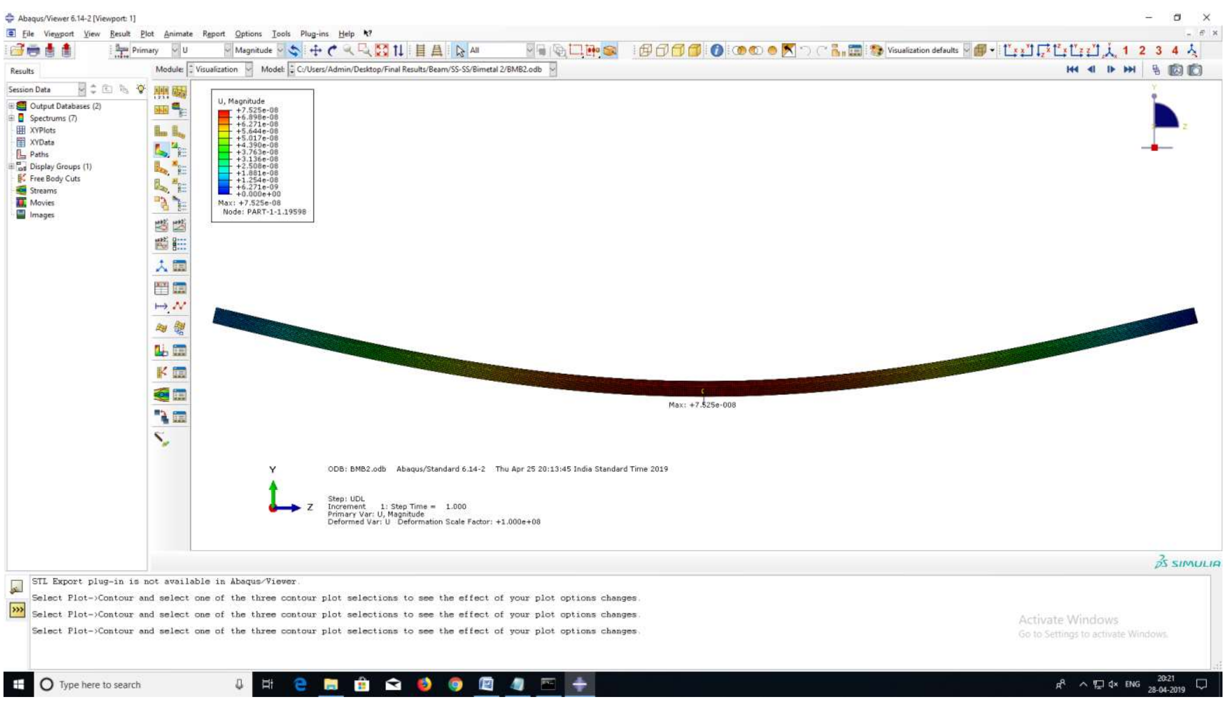
E.g., Examples 1.1 and 1.2 with applied simply supported boundary conditions on one of the beam edge

10.Results and Discussion

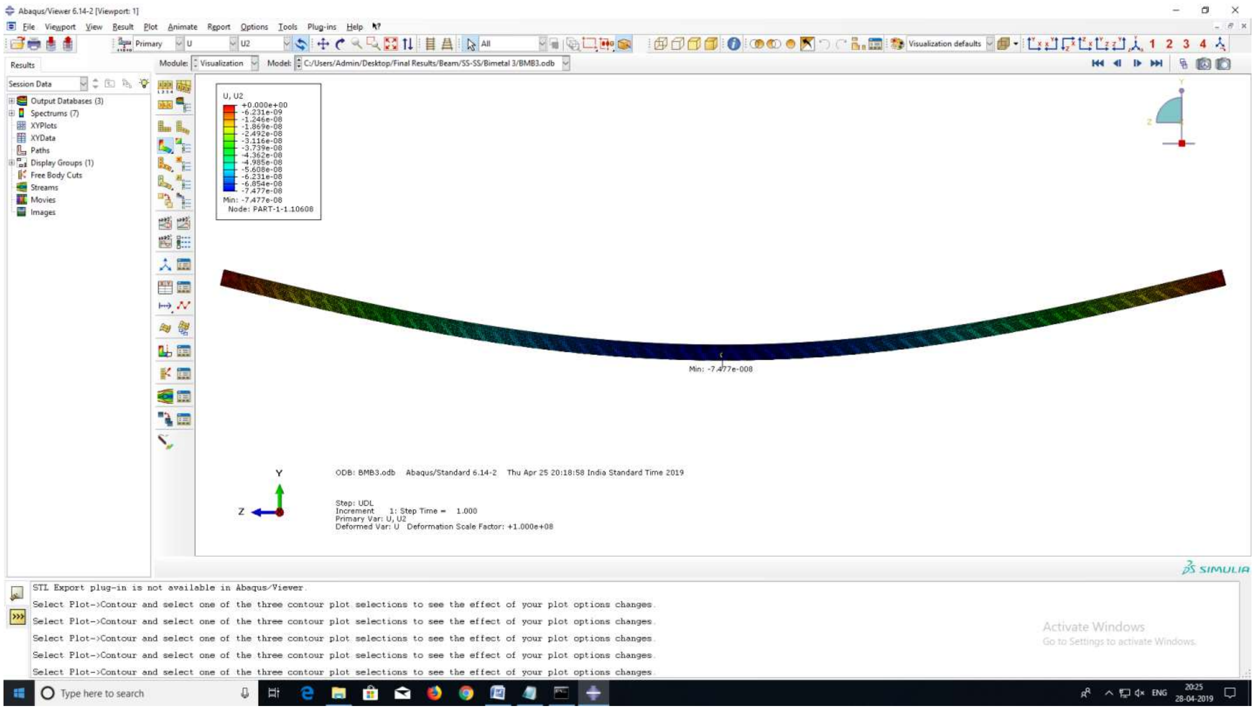
Screenshots presented below depict maximum deflection for simply supported beams (examples 1.1, 1.2.1, 1.2.2 and 1.2.3) and simply supported plates (examples 2.1, 2.2.1, 2.2.2 and 2.2.3) under the action of UDL.



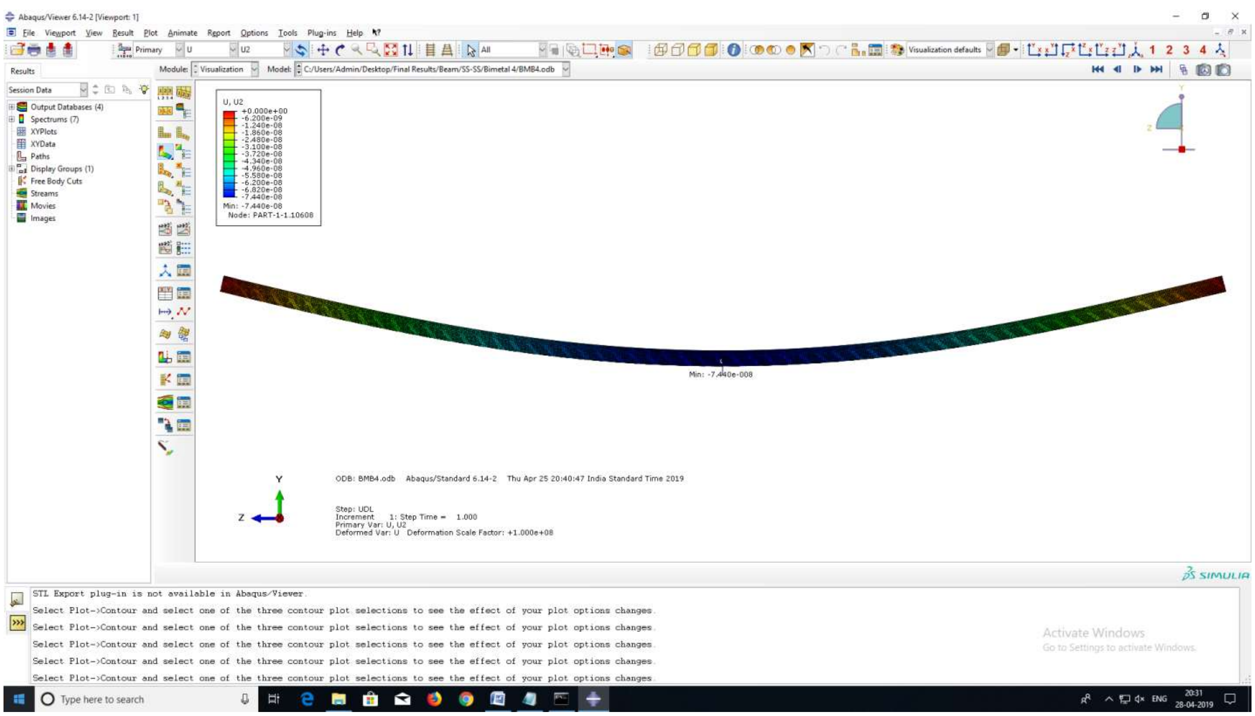
Examples 1.1 with maximum deflection of 7.524e-8 m



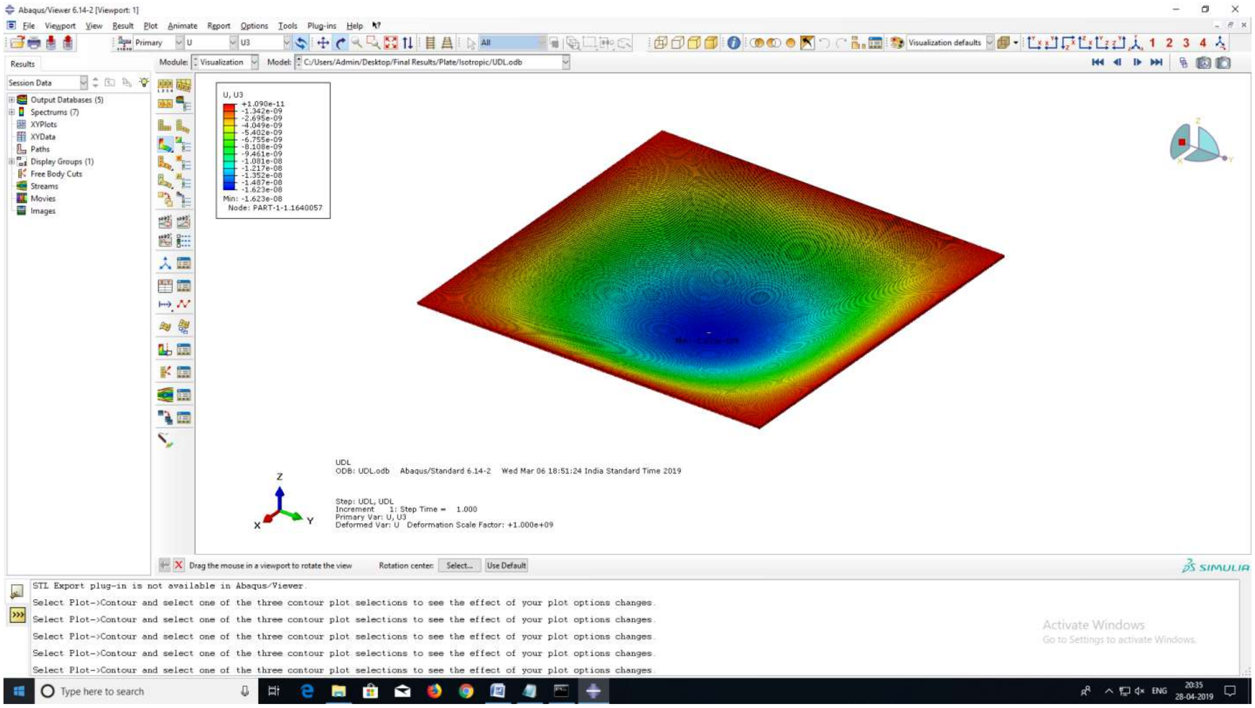
Examples 1.2.1 with maximum deflection of 7.525e-8 m



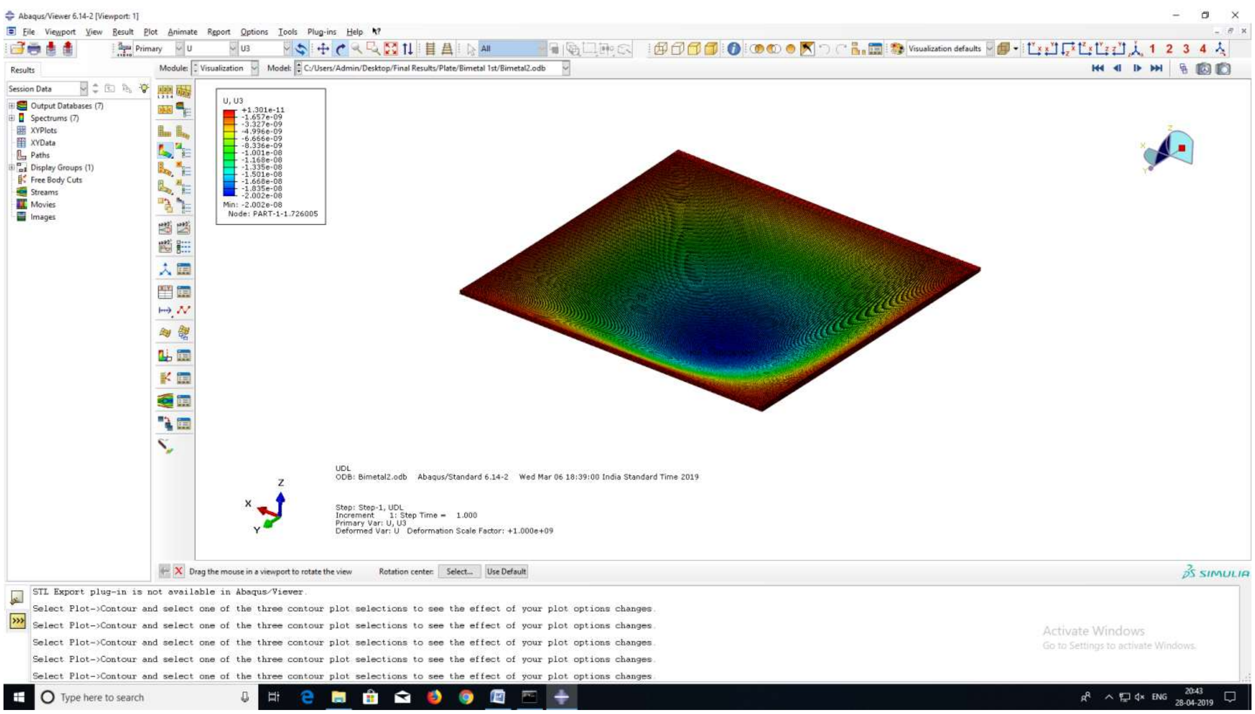
Examples 1.2.2 with maximum deflection of 7.477e-8 m



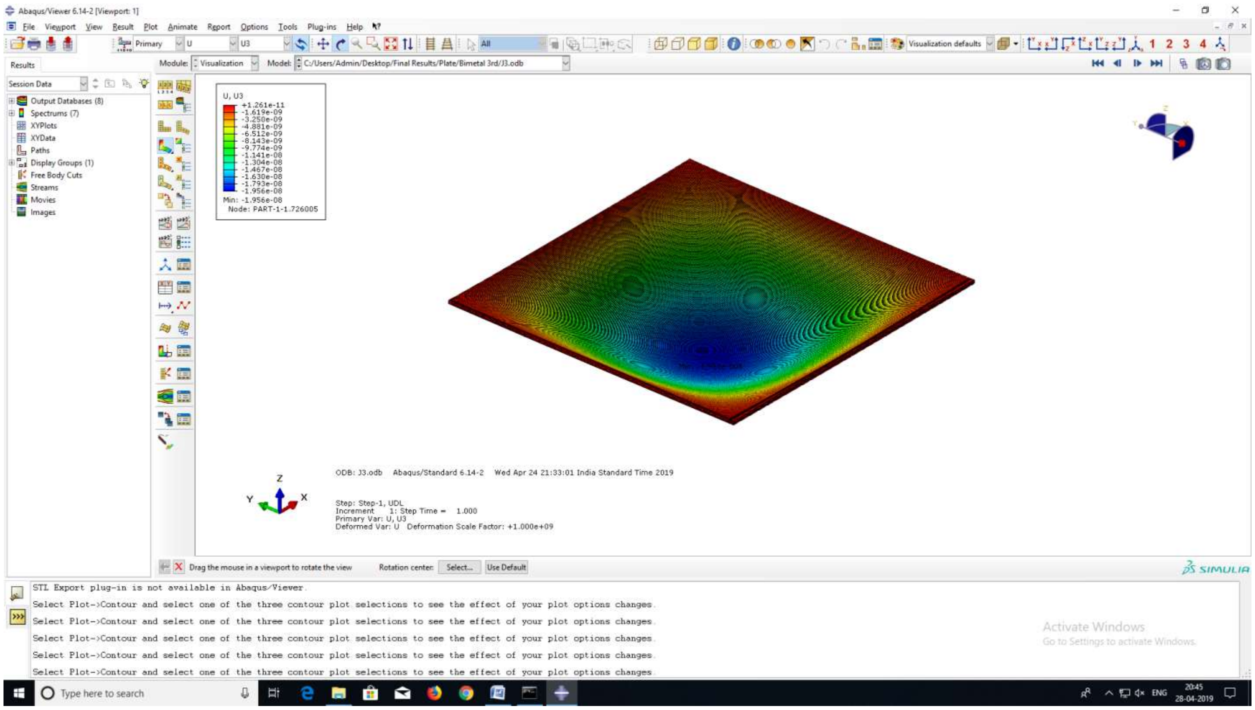
Examples 1.2.3 with maximum deflection of 7.440e-8 m



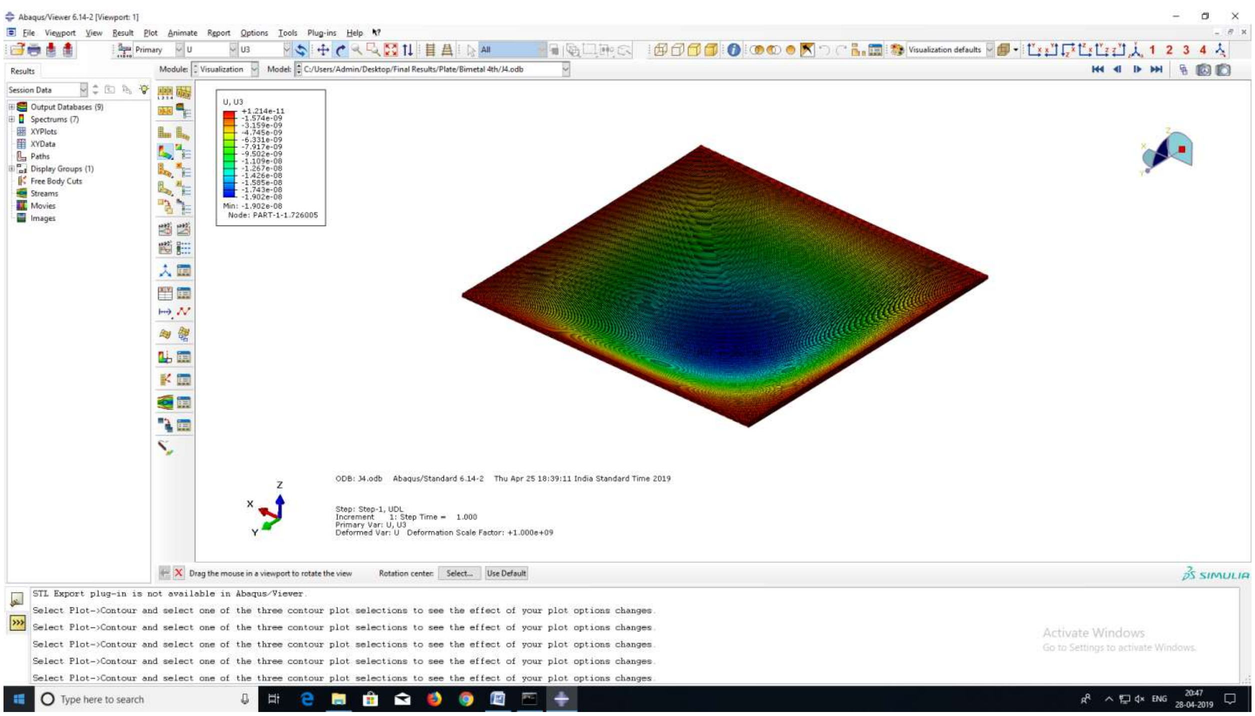
Examples 2.1 with maximum deflection of 1.623e-8 m



Examples 2.2.1 with maximum deflection of 2.002e-8 m



Examples 2.2.2 with maximum deflection of 1.956e-8 m



Examples 2.2.3 with maximum deflection of 1.902e-8 m

For examples 1.1, 1.2.1, 1.2.2 and 1.2.3

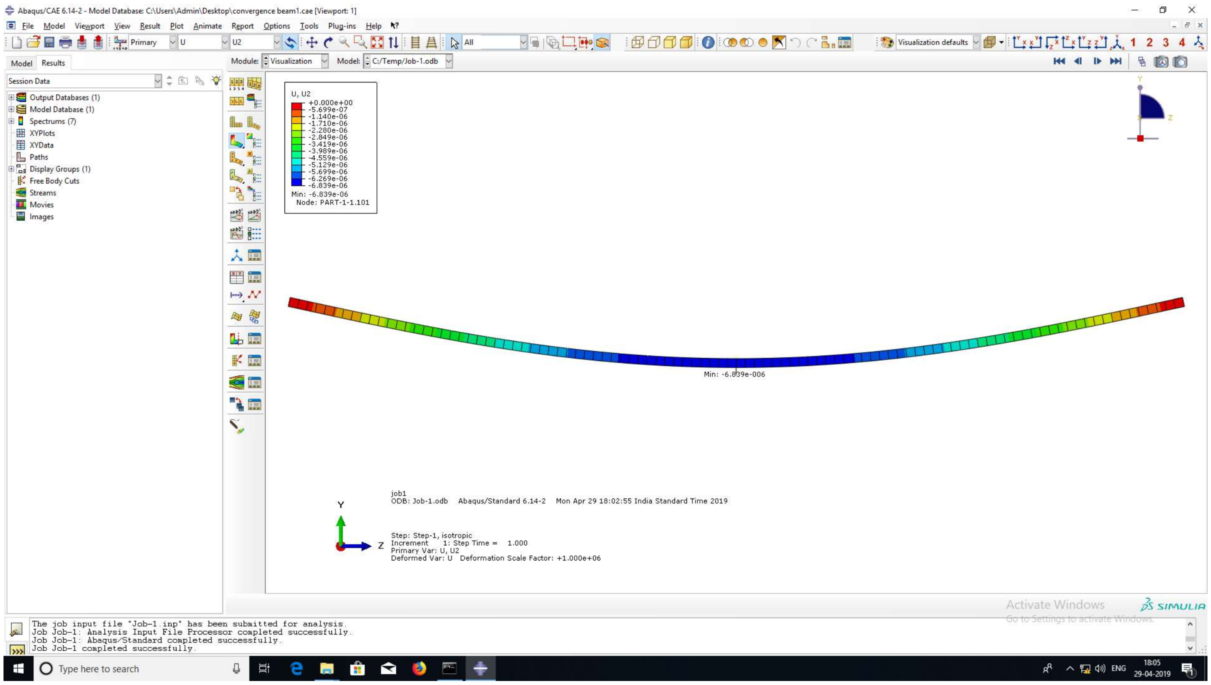
Maximum non-dimensional beam transverse displacement $\left(\frac{\mathrm{w}_{\mathrm{max}}\, E.I}{\mathrm{q_o}.\,\mathrm{L}^4}\right)$			
h/L	**C_{10} (MPa)**	**Abaqus FEA**	**Literature (Ref. [1])**
0.01	-	0.013167	0.0132
0.01	10	0.013169	-
0.01	50	0.013085	-
0.01	100	0.013020	-

For examples 2.1, 2.2.1, 2.2.2 and 2.2.3

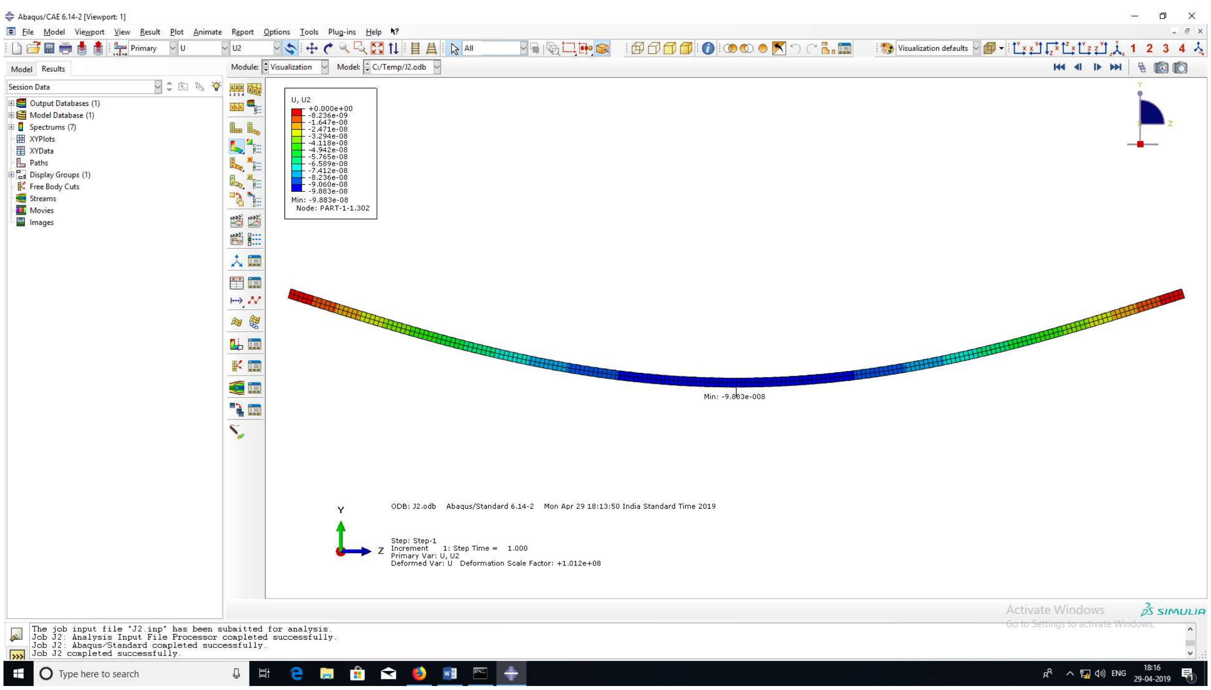
Maximum non-dimensional plate transverse displacement $\left(\frac{\mathrm{w}_{\mathrm{max}}\, D}{\mathrm{q_o}.\,\mathrm{L}^4}\right)$			
h/L	**C_{10} (MPa)**	**Abaqus FEA**	**Literature (Ref. [2])**
0.01	-	0.004287	0.00406
0.01	10	0.005288	-
0.01	50	0.005167	-
0.01	100	0.005024	-

11.Convergence Study

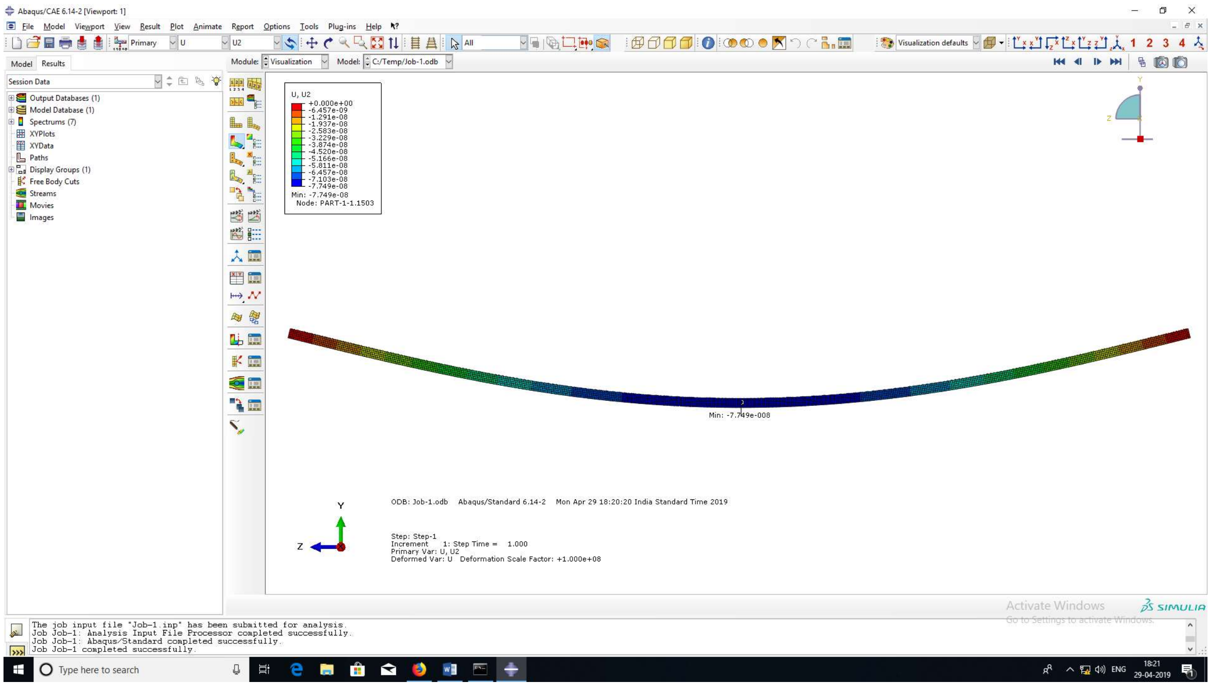
Convergence study is performed for example 1.1. The obtained optimum value of element size from this study is then utilized for examples 1.2, 2.1 and 2.2.



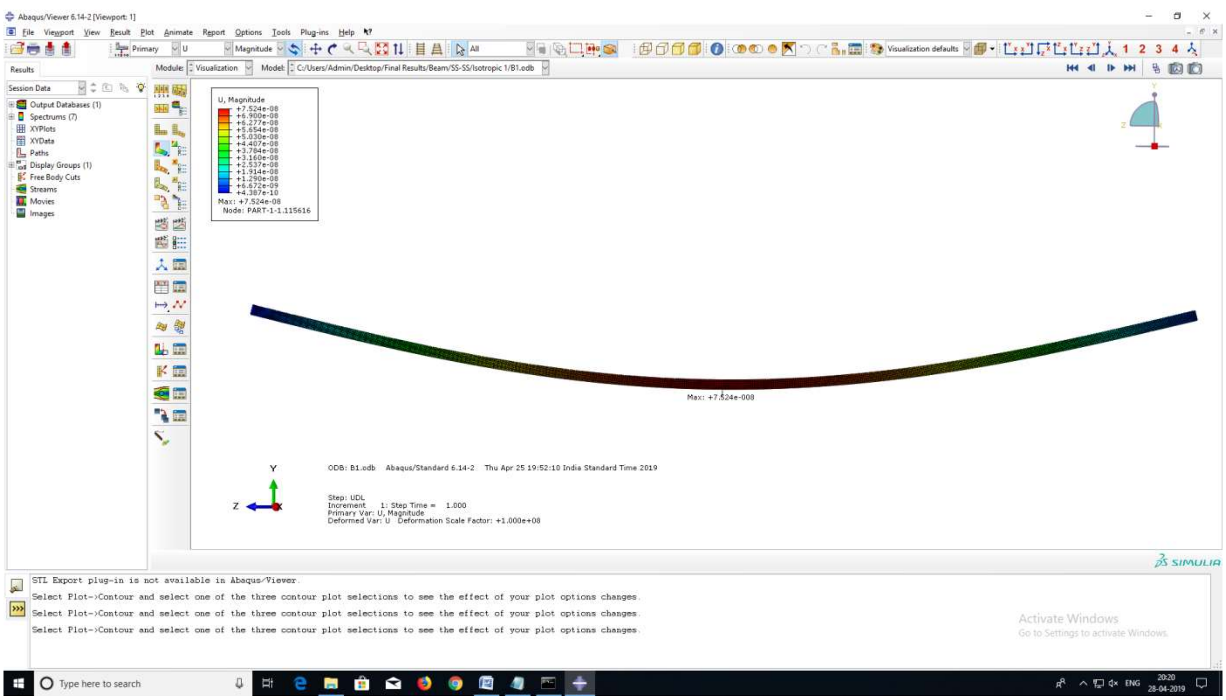
Examples 1.1 with one element on beam cross-section



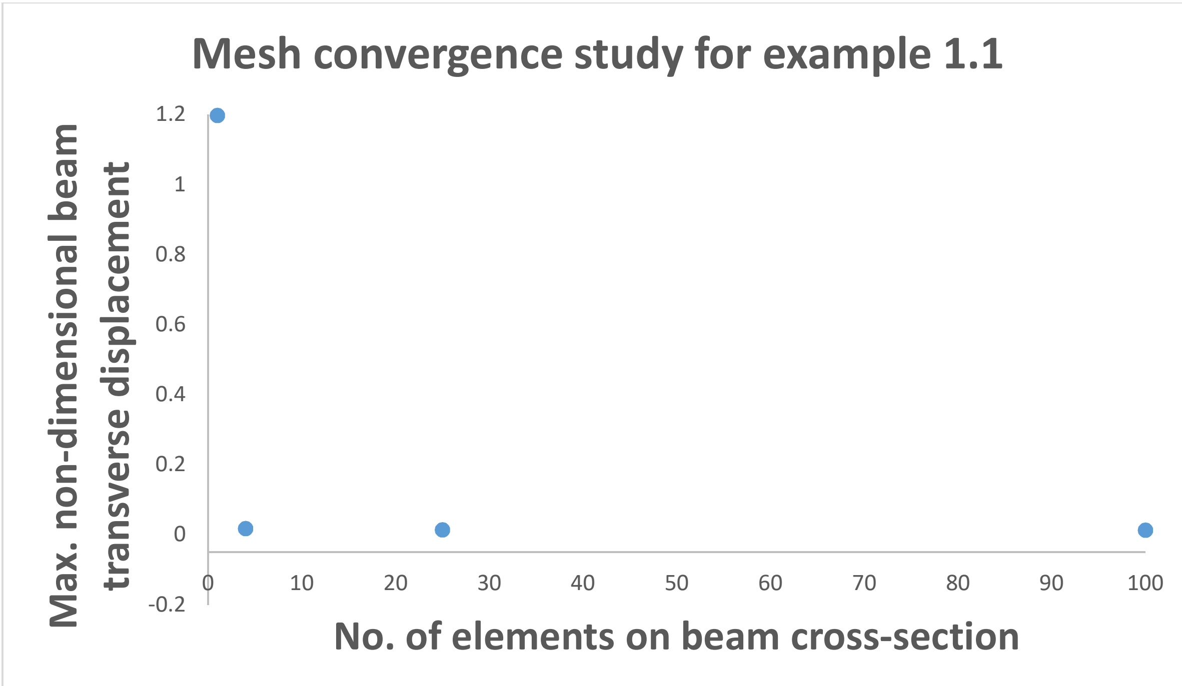
Examples 1.1 with four elements on beam cross-section



Examples 1.1 with twenty five elements on beam cross-section



Examples 1.1 with one hundred elements on beam cross-section


Mesh convergence study for example 1.1
Max. non-dimensional beam transverse displacement
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
10
20
30
40
50
60
70
80
90
100
No. of elements on beam cross-section

12. Conclusions

1. For examples 1.1 and 2.1 with beam and plate made of linear, elastic, homogenous isotropic material; obtained values of maximum non-dimensional transverse displacements using 3D FEA match with corresponding analytical results reported in the literature.
2. For examples 1.2 and 2.2 with beam and plate made of bi-material, obtained values of maximum non-dimensional transverse displacement using 3D FEA match to some extent with corresponding results obtained for examples 1.1 and 2.1. Reasons for the same are as follows:
   - 2.1 Layer of Neo-Hookean material added on isotropic material has lower value of C_{10} as compared to Young's modulus of isotropic material.
   - 2.2 This leads to very small increase in stiffness of examples 1.2 and 2.2 as compared to that of examples 1.1 and 2.1.

13. References

1. Shimpi, R. P., Shetty, R. A., & Guha, A. (2017). A simple single variable shear deformation theory for a rectangular beam. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, *231*(24), 4576-4591.

2. Shimpi, R. P., Guruprasad, P. J., & Pakhare, K. S. (2018). Single variable new first-order shear deformation theory for isotropic plates. *Latin American Journal of Solids and Structures*, *15*(10).